Homework 9: Neural Networks and the Perceptron

1. You are given a perceptron with:

$$w_1 = 1, \quad w_2 = -2, \quad w_3 = 3, \quad \beta = -2$$

For each input below:

- Compute the linear combination $z = w_1x_1 + w_2x_2 + w_3x_3 + \beta$
- Apply three activation functions:
  - **Unit step:** $f(z) = \begin{cases} 1 & z \geq 0 \\ 0 & z < 0 \end{cases}$
  - **Sigmoid:** $f(z) = \frac{1}{1+e^{-z}}$
  - **ReLU:** $f(z) = \max(0, z)$

Input A: $(0, 0, 0)$

$z =$ _____ Unit Step: _____ Sigmoid: _____ ReLU: _____

Input B: $(3, 2, 2)$

$z =$ _____ Unit Step: _____ Sigmoid: _____ ReLU: _____

Input C: $(1, 4, 1)$

$z =$ _____ Unit Step: _____ Sigmoid: _____ ReLU: _____

Input D: $(2, 0, 3)$

$z =$ _____ Unit Step: _____ Sigmoid: _____ ReLU: _____

2. Given the perceptron with:

$$w_1 = 1, \quad w_2 = -2, \quad w_3 = 3, \quad \beta = -10$$

Use the **unit step function** to compute the output for each row in the dataset below.

x_1	x_2	x_3	y (true)	z (show work)	Prediction
0	0	0	1	_____	_____
3	2	2	0	_____	_____
1	4	1	0	_____	_____
2	0	3	1	_____	_____

Accuracy Calculation:

Number of correct predictions: _____ Total examples: 4 $\Rightarrow$ Accuracy: _____

3. You may find it helpful to return to the slide decks from earlier in the course on bias.

(a) How does changing the **bias** affect how a perceptron classifies data?

(b) How does changing one of the **weights** affect the classification or decision boundary?

4. You are given a neural network with the following structure:

- Input layer: 3 features
- Hidden Layer 1: 4 neurons, sigmoid activation
- Hidden Layer 2: 2 neurons, sigmoid activation
- Output layer: 1 neuron, unit step activation

(a) Draw the network below. Include all layers, neurons, and connections.

(b) Compute the total number of trainable parameters (weights and biases)

- Input $\rightarrow$ Hidden Layer 1: # weights: _____ # biases: _____
- Hidden Layer 1 $\rightarrow$ Hidden Layer 2: # weights: _____ # biases: _____
- Hidden Layer 2 $\rightarrow$ Output Layer: # weights: _____ # biases: _____
- **Total parameters:** _____